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Development of Number Line Representations in Children With Mathematical Learning Disability

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Children with a mathematical learning disability (MLD, n = 19) and low achieving (LA, n = 43) children were identified using mathematics achievement scores below the 11th percentile and between the 11th and 25th percentiles, respectively. A control group of typically achieving (TA, n = 50) children was also identified. Number line and speed of processing tasks were administered in 1st and 2nd grade and a working memory battery in 1st grade. In both grades, the MLD children were less accurate in their number line placements and more reliant on a natural number-magnitude representational system to make these placements than were TA children. The TA children were more reliant on the school-taught linear system in both grades. The performance of the LA children was similar to that of the MLD children in first grade and to the TA children in second. The central executive component of working memory contributed to across-grade improvements in number line performance and to group differences in this performance.

Several large-scale population-based, prospective studies and a number of smaller-scale studies have consistently found that between 5% and 10% of children and adolescents will experience a substantive learning deficit—not attributable to low cognitive ability—in at least one area of mathematics before graduating from high school (Badian, 1983; Barbaresi, Katusic, Colligan, Weaver, & Jacobsen 2005; Ostad, 1998; Shalev, 2007; Shalev, Manor, & Gross-Tsur, 2005). These individu-

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als are considered to have a mathematical learning disability (MLD), and are joined by another 5% and perhaps many more children and adolescents who experience more mild and circumscribed learning difficulties in mathematics (for recent reviews see Berch & Mazzocco, 2007). The mathematics achievement of this latter group of low achieving children (LA) is below expectations based on their cognitive ability and reading achievement, and the mechanisms contributing to their difficulties with mathematics learning may differ from those underlying MLD (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Murphy, Mazzocco, Hanich, & Early, 2007). The central executive component of working memory has been implicated as a core mechanism underlying differences in the mathematical cognition of children with MLD but not their LA peers, but this remains to be confirmed. Moreover, the mathematical areas in which these groups may be similar or different are not well understood.

Children with MLD have deficits in a wide range of basic mathematical domains, including a delayed understanding of counting concepts (Geary, Bow-Thomas, & Yao, 1992), difficulties remembering arithmetic facts (Geary, 1993; Jordan, Hanich, & Kaplan, 2003; Jordan & Montani, 1997), and poor conceptual knowledge of rational numbers (Mazzocco & Devlin, in press). The delayed learning of LA children, in contrast, may center on basic numerical representations, including the number line (Geary et al., 2007). The number line is a core mathematical tool that is involved in the representation of natural counting numbers to the coordinate system in geometry and algebra, among many other uses (Case et al., 1996; Griffin, Case, & Siegler, 1994; National Council of Teachers of Mathematics, 2006). Difficulties in the development of a mathematically accurate representation of the number line have implications for mathematics learning through high school and beyond. In first grade, there are similarities in the number line representations of LA children and children with MLD; specifically, in comparison to typically achieving (TA) children, they are less accurate in their placement of numbers on the line and delayed in the development of a linear, mathematical representation of the number line (Geary et al., 2007). Little else is known about the similarities and differences in the development of number line representations of children with MLD and LA children or the underlying mechanisms that may contribute to these similarities and differences. The current longitudinal study sought to address this gap by assessing the development of the number line representational system across first and second grade and by testing hypotheses regarding potential working memory mechanisms that may contribute to group differences in the development of this system.

NUMBER LINE REPRESENTATIONS

Learning the mathematical number line is not only a core element of basic education in mathematics (National Council of Teachers of Mathematics, 2006); it is an area of study in cognitive psychology (Opfer & Siegler, 2007; Siegler & Opfer, 2003; Siegler & Booth, 2004) and in cognitive neuroscience (Zorzi, Priftis, & Umiltá, 2002). Educationally, individual differences in children's learning of the linear, mathematical number line are correlated with mathematics achievement in all grades in which it has been assessed (Booth & Siegler, 2006). Theoretically, it is an area of interest because magnitude representations, including those that support the number line, may be based on a potentially inherent number-magnitude system that is supported by specific areas in the parietal cortices of the left- and especially the right-hemisphere and because damage to one or both of these regions can result in mathematical deficits (Isaacs, Edmonds, Lucas, & Gadian, 2001; Kadosh et al., 2007; Molko et al., 2003), including disruption of the ability to represent the number line spatially (Zorzi et al., 2002).

Making placements on a physical number line that are based on use of the inherent number-magnitude system results in a pattern that conforms to the natural logarithm (*ln*) of the number (Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 1992; Siegler & Opfer, 2003). In other words, use of this natural representational system results in placements that are compressed for larger magnitudes such that the perceived distance between 89 and 90 is smaller than the perceived distance between 2 and 3. If children with MLD have deficits in the number-magnitude representational system, then their number line placements might not conform to the natural log model or might show less precision than other children when they make placements using this representation; specifically, more compression (closer placements) for smaller numbers (Geary et al., 2007).

With schooling, TA children's number line placements gradually conform to the linear mathematical system (Siegler & Booth, 2004); the difference between two consecutive numbers is identical regardless of position on the number line. As we describe later, the central executive is the hypothesized mechanism that enables the creation of a linear mental representation of the number line, potentially through modification of the natural number-magnitude system (Geary, 2007a, 2007b). If children with MLD do not show evidence of a developing linear representational system, then another source of MLD might be difficulty in modifying the natural system to conform to the school-taught linear system.

WORKING MEMORY

Working memory is the ability to hold a mental representation of information in mind while simultaneously engaging in other mental processes. As noted, working memory is composed of a central executive expressed as attention-driven control of information represented in two core representational systems (Baddeley, 1986). The systems are a language-based phonetic buffer, and a visuospatial sketch pad. The central executive is important in the initial stages of academic learning, that is, for the acquisition of novel school-taught competencies (e.g., linear number line) and suppression of more natural modes of understanding the presented information (e.g., use of the natural number-magnitude system; Geary, 2007a, 2007b). The central executive has also been implicated as a core mechanism contributing to the learning deficits of children with MLD (Geary et al., 2007; McLean & Hitch, 1999; Swanson, 1993; Swanson & Sachse-Lee, 2001). The visuospatial sketch pad is of interest, because the parietal areas associated with number and magnitude processing are situated near brain regions that support aspects of visuospatial processing and because damage to these parietal regions disrupts the ability to form spatial representations and to imagine a mental number line (Zorzi et al., 2002).

The potential contributions of working memory to number line learning are complicated by speed of processing. The issues center on whether individual differences in working memory are driven by more fundamental differences in speed of neural processing (Kail, 1991), or whether the attentional focus associated with the central executive speeds information processing (Engle, Tuholski, Laughlin, & Conway, 1999). In any case, a systematic assessment of the potential mechanisms underlying group differences on the number line task requires simultaneous measurement of working memory and speed of processing. This is because in addition to deficits on measures of working memory, children with MLD process information more slowly than TA children (Bull & Johnston, 1997; Swanson & Sachse-Lee, 2001).

CURRENT STUDY

The current study is one of the few investigations of number line development in children with MLD and their LA peers and the first longitudinal study in this area. Our goal was to assess group similarities and differences in accuracy of number line placements and in the form of the underlying representational systems (i.e., natural number-magnitude or linear; Siegler & Opfer, 2003) across first and second grade and to compare and contrast these to the number line placements and representations of same-grade TA children. We also tested the hypothesis that individual differences in number line representations will be influenced by individual differences in visuospatial working memory and that individual differences in across-grade change from use of the natural representational system to the linear system will be influenced by individual differences in the central executive (Geary, 2007a; Menon, Rivera, White, Glover, & Reiss, 2000). Finally, we assessed the within and across grade relation between group differences in working memory and group differences in accuracy of number line placements and in the form of the underlying mental representations of the number line.

METHOD

Participants

All kindergarten children from 12 elementary schools were invited to participate in a longitudinal prospective study of MLD (Geary et al., 2007). Parental consent and child assent were received for 311 (37%) of these children We classified children as MLD if their scores for the mathematical achievement test (described later) were less than the 11th national percentile (based on test norms) in both first and second grade, and if their IQ scores were between 80 and 130—the range was restricted because the relation between IQ and MLD is not known. Of the 261 children with IQ scores in this range, 19 (7.3%, 9 male) met the MLD criteria. A sample of 43 (16 male) LA children with scores between the 11th and 25th percentile, inclusive, on the mathematics test in both grades was also identified, and provided a comparison group similar to samples identified as MLD in many previous studies (Murphy et al., 2007). The sample of 50 (24 male) TA children had mathematics scores between the 26th and 74th percentiles, inclusive, in both grades.

At the time of the administration of the achievement tests in 1st grade, the mean ages of the children in the MLD, LA, and TA groups were 85 ($SD = 3$), 85 ($SD = 4$), and 87 ($SD = 4$) months, respectively ($p > .05$). There were neither differences in the number of boys and girls across groups, $\chi^2(2) = 1.2$, $p > .50$, nor differences ($p > .20$) in the ethnic distribution of the children across groups (65% white, and most of the remaining children were black or Asian).

Standardized Measures

Intelligence. In kindergarten, the children were administered the Raven's Coloured Progressive Matrices (Raven, Court, & Raven, 1993), a non-timed test that is considered to be an excellent measure of fluid intelligence. A percentile ranking was obtained for each child and these were converted to IQ scores standardized with a mean of 100 and an *SD* of 15. In first grade, the children were administered the Vocabulary and Matrix Reasoning subtests of the Wechsler Abbreviated Scale of Intelligence (Wechsler, 1999). The latter provide age-referenced standard IQ scores with a mean of 100 and SD of 15. The Raven and Wechsler scores were significantly correlated, $r(110) = .42$, $p < .0001$, and the final IQ score was the mean of these two measures.

Achievement. The children were administered the Numerical Operations and Word Reading subtests from the Wechsler Individual Achievement Test-II–Abbreviated (Wechsler, 2001). The Numerical Operations subtest assesses number discrimination, rote counting, number production, basic addition and subtraction, multi-digit addition and subtraction, and some multiplication and division. The Word Reading subtest includes matching and identifying letters, rhyming, beginning and ending sounds, phoneme blending, letter sounds, and word recognition. For each test, we used the age-based national percentile ranking.

Working Memory

The Working Memory Test Battery for Children (WMTB–C; Pickering & Gathercole, 2001) consists of nine subtests that assess the central executive, phonological loop, and visuospatial sketchpad. All of the subtests have six items at span levels ranging from one to six to one to nine. Passing four items at a level moves the child to the next level. At each span level, the number of items (e.g., words) to be remembered is increased by one. Failing three items at a given span level terminates the subtest. The order of subtests was designed so as not to over-tax any one component area of working memory and was generally arranged from easiest to hardest: Digit Recall, Word List Matching, Word List Recall, Nonword List Recall, Block Recall, Mazes Memory, Listening Recall, Counting Recall, and Backward Digit Recall. Standard scores are determined for each subtest and for the three component areas.

Central executive. The central executive is assessed using three dual-task subtests. Listening Recall requires the child to determine if a sentence is true or false, and then recall the last word in a series of sentences. Counting Recall requires the child to count a set of 4, 5, 6, or 7 dots on a card, and then to recall the number of counted dots at the end of a series of cards. Backward Digit Recall is a standard format backward digit span.

Phonological loop. Digit Recall, Word List Recall, and Nonword List Recall are standard span tasks with variant stimuli; the child's task is to repeat, verbatim, a string of words spoken by the experimenter. In the Word List Matching task, a series of words, beginning with two words and adding one word at each successive level, is presented to the child. The same words, but possibly in a different order, are then presented again, and the child's task is to determine if the second list is in the same or different order than the first list.

Visuospatial sketch pad. Block Recall is another span task, but the stimuli consist of a board with nine raised blocks in what appears to the child as a "random" arrangement. The blocks have numbers on one side that can only be seen from the experimenter's perspective. The experimenter taps a series of blocks, and the child's task is to duplicate the tapping in the same order as presented by the experimenter. In the Mazes Memory task, the child is presented a maze with more than one solution, and a picture of an identical maze with a path drawn for one solution. The picture is removed and the child's task is to duplicate in the path in the response booklet. At each level, the mazes get larger by one wall.

Speed of processing. Using the same stimuli as Mazzocco and Myers (2003), two Rapid Automatized Naming (RAN; Denckla & Rudel, 1976) tasks were used to assess processing speed. The child was presented with five letters or numbers to first determine if the child could read the stimuli correctly. After these practice items, the child was presented with a 5 × 10 matrix of fifty incidences of these same letters or numbers, and was asked to name them as quickly as possible without making any mistakes. Reaction time (RT) was measured via a stopwatch. For each type of stimulus, the task generated an RT, number correct, and number of reversals for letters (b and d, p and q). Errors and reversals were infrequent and thus only raw RTs were used from this test.

Number line task. Stimuli for this task were twenty-four 25 cm number lines. In first grade, the stimuli were printed across the center of a standard 8 1/2" × 11" paper in a landscape orientation, and in second grade the stimuli were presented in the same format but on the screen of a laptop computer. Each number line had a start point of 0 and an endpoint of 100 with a target number printed approximately 5 cm above it in a large font (72 pt). Following Siegler and Booth (2004, experiment 1), target numbers were 3, 4, 6, 8, 12, 17, 21, 23, 25, 29, 33, 39, 43, 48, 52, 57, 61, 64, 72, 79, 81, 84, 90, and 96. The numbers below 30 were over-sampled to allow for fitting of a natural log model of the children's placements. Experimental stimuli were presented in a random order for each child.

The child was first presented with a number line that included the 0 and 100 endpoints and marked in increments of 10. After discussion about number lines and when it was determined that the child recognized the concept, a blank number line, containing only the endpoints 0 and 100, was presented and the child was asked to determine where the number 50 should go. In first grade, the child was instructed to mark across the line with a pencil where 50 should fall, and in second grade the child was given the same instructions but used a mouse to mark the number line on the computer screen. A number line either printed on paper (first grade) or presented on the computer screen (second grade) with the endpoints and the location of 50 marked was then shown to the child. To ensure the child understood the task, the child's response was compared to the printed version and the experimenter discussed with the child how "50 is half of 100, so it goes half way between 0 and 100." The experimental trials were then administered. The trials began with "If this is zero (pointing) and this is 100 (pointing), where would N go?" The sentence was repeated for each trial until child was comfortable with the procedure.

As described in Results, the task yields a score for overall degree of error (i.e., difference between the correct placement and the actual placement averaged across items), as well as individual scores for percentage of trails on which the child used a linear or *ln* strategy to make the placements and scores for absolute degree of error for each of these two strategies. Group-level median values are also used for each item to make inferences about the modal representational system (i.e., linear or *ln*) used by children in the group (Siegler & Opfer, 2003).

Procedure

All children were tested in the spring of their kindergarten year and in the fall and spring of first and second grade. The spring assessments included the achievement tests in both grades and the Progressive Matrices in kindergarten and the Vocabulary and Matrix Reasoning tests in first grade. The first grade fall testing included the RAN RT and number line task; see Geary et al. (2007) for a complete description of the tasks used in the overall study. For second grade, the number line task was moved to the spring due to scheduling constraints. The majority of children were tested in a quiet location at their school site, and occasionally in a testing room on the university campus or in a mobile testing van if the child moved between assessments. Each of these testing sessions required about 40 min. The WMTB-C was added to the study in the summer following kindergarten and we received parental permission to assess 270 children from the original sample; this included 18 of the 19 MLD children, 39 of the 43 LA children, and 46 of the 50 TA children. For the majority of children, the battery was administered in the testing van during first grade. The assessment required about 60 min and occurred when the child was not in school (e.g., weekend). Across groups, the mean age at the time of administration of the WMTB-C ranged between 83 and 85 months and did not differ across groups ($p > .10$).

RESULTS

In the first section, we present group differences on the standardized IQ and achievement tests. In the second and third respective sections, we present results for the working memory and speed of processing variables, and the number line measures. In these sections, significant F ratios for group differences are followed by least squares pair-wise t-tests, controlling for IQ or other variables (described later). For significant group differences, effect sizes (d) were calculated as $(M_1 - M_2)/SD$; SD was estimated across all participants, and M_1 was the mean of the lower achieving group. In the fourth section, we analyze the relations between individual differences in working memory and speed of processing and individual differences on the number line measures, and in the final section we present analyses of potential mediators of group differences on the latter measures.

Standardized Tests

Group differences in IQ (Table 1) were significant, $F(1,102) = 10.34$, $p < .001$; follow-up HSD *t*-tests indicated the mean for the TA group was higher than that of the two other groups (*p*s < .05), that in turn did not differ ($p > .05$). Because of the significant group differences, IQ was used as a covariate in all subsequent analyses. In first and second grade, the Numerical Operations scores (i.e., percentile ranking) of the MLD group were, by design, significantly lower than those of the LA (*d*s = –.78, –.79, respectively) and TA (*d*s = –2.28, –2.26) groups; the two latter groups differed as well (*d*s = –1.50, –1.47; *p*s < .001). In first and second grade, the Word Reading scores of the MLD group were significantly (*p*s < .005) lower than those of the LA (*d*s = –.87, –.90) and TA (*d*s = –1.40, –1.38) groups; the advantage of the TA group relative to the LA group was significant in first grade ($d = -.53$, $p < .05$), but not second ($p > .05$).

Working Memory and Speed of Processing

For the age at which the WMTB–C was administered to the children in our study, our overall sample of 270 children was substantially larger than the standardization sub-sample of 86 children (Pickering & Gathercole, 2001). We thus standardized ($M = 100$, $SD = 15$) raw scores for the Phonological Loop, Visuospatial Sketch Pad, and Central Executive scales for our overall sample and used these for subsequent analyses. The associated mean standard scores are shown in Table 2. Group differences were significant for the phonological loop, $F(2, 99) = 4.84$, $p < .01$, and the central executive, $F(2, 99) = 7.35$, $p < .005$, but not for the visuospatial sketch pad ($p >$ 05). Follow-up *t*-tests revealed the mean phonological loop scores of the TA group were significantly (*p*s < .05) higher than those of the MLD ($d = -1.13$) and LA groups ($d = -.56$); the two latter groups did not differ ($p > .10$). In contrast, the mean central executive scores of the MLD group were significantly (*p*s < .01)

TABLE 1
Standardized Intelligence and Percentile Rankings for Achievement Tests

				First Grade				*Second Grade*			
		IQ		*Word Reading*		*Numerical Operations*		*Word Reading*		*Numerical Operations*	
	N	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
MLD	19	96	9	31	24	5	1	32	29	5	3
LA	43	101	9	57	28	19	6	58	28	20	4
TA	50	107	10	73	25	46	10	72	22	48	12

Note. MLD = math disabled, LA = low achieving, and TA = typically achieving.

TABLE 2
Standardized Working Memory Test Battery Scores and Raw Speed of Processing RTs

	Working Memory System						*Speed of Processing RAN RTs*			
	Phonological Loop		*Visuospatial Sketch Pad*		*Central Executive*		*1st Grade*		*2nd Grade*	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
MLD	88	17	90	14	87	13	53	17	40	15
LA	97	14	101	14	98	10	43	12	34	11
TA	106	14	101	12	104	13	38	9	32	5

Note. MLD = math disabled, LA = low achieving, and TA = typically achieving. The working memory scores are standardized with $M = 100$, $SD = 15$. RAN RTs are the mean (sec) for the letter naming and number naming measures.

lower that those of the LA ($d = -.85$) and TA ($d = -1.31$) groups; the two latter groups did not differ ($p > .05$).

The RAN RT measure was the mean raw RT for the letter and number naming tasks. Group differences for were significant in first grade, $F(2, 108) = 6.36$, $p < .005$, and a trend emerged for second grade, $F(2, 108) = 3.06$, $p = .051$. The RTs of the MLD group were significantly (*p*s < .02) longer than those of the LA ($d = .77$) and TA ($d = 1.15$) groups in first grade, and longer than those of the TA group in second grade ($d = .80$). No other differences were significant (*p*s > .05).

Number Line Placements

We used three ways to analyze the accuracy of number line placements and assess underlying representational systems: Overall degree of error, goodness of fit of *ln* and linear models to the group-level median placements, and individual-level trial-by-trial use of *ln* and linear strategies for making the placements. For all of the corresponding analyses, the same-grade Word Reading percentile ranking scores and IQ were used as covariates. The first approach provides an overall assessment of accuracy (or degree of error) of the placements, and the two latter approaches provide methods that allowed us to make inferences about the forms of mental representation the children were using to make the placements.

Error. The overall degree of error of the placement is simply the mean difference (across items) between the actual position of the number on the number line and the child's estimate of this position. In first grade, the TA children's estimates differed from the correct position by an average of 12 ($SD = 7$), as compared to mean differences of 18 (*SD* = 7) and 25 (*SD* = 8) for the LA (d = .71) and MLD (d = 1.53) groups, respectively; d = .82 for the MLD versus LA difference. In second grade, TA children's estimates differed from the correct position by an average of 6 (*SD* = 2), as compared to mean differences of 9 (*SD* = 4) and 15 (*SD* = 8) for the LA (d = .57) and MLD (d = 1.70) groups, respectively; d = 1.13 for the MLD versus LA difference. A mixed ANCOVA, with group as a between subject factor and grade as a within subjects factor, confirmed significant group differences, $F(2, 106) = 10.59$, $p < .0001$, and improved accuracy across grades, $F(2, 106) = 7.19$, $p < .01$, but also revealed a non-significant interaction ($p > .25$). Follow-up analyses confirmed an overall group difference in first, $F(2, 107) = 6.14$, $p < .005$, and second grade, $F(2, 107) = 8.98$, $p < .005$. All of the pair-wise t-tests were significant in first grade (ps < .05). In second grade, the estimates of TA and LA groups did not differ ($p > .25$), but the placements of the MLD group were less accurate than those of the two other groups (ps < .005).

To determine if degree of error varied across the first and second ½ of the number line, the 14 numbers <50 and the 10 > 50 were split. Across all participants, the degree of error was 18 for the first ½ and 14 for the second ½ in first grade, and 8 and 9, respectively, in second. The correlation between the first ½ and second ½ was .55 in first grade and .66 in second (ps < .0001); these values produce respective alphas of .69 and .79. A two (grade) by two (split) by three (group) mixed ANCOVA, with IQ and first and second grade reading scores serving as covariates, and split as a within subjects factor confirmed the group differences in degree of error, $F(2, 106) = 12.58$, $p < .0001$, and the across-grade reduction in degree of error, $F(1, 106) = 9.87$, $p < .005$. The main effect for split was not significant, $F(1, 106) < 1$, nor were the split by group, $F(2, 106) < 1$, split by grade, $F(1, 106) = 2.94$, $p > .05$, or split by grade by group, $F(2, 106) < 1$, interactions.

Representational system. Whereas the first method provides useful information on degree of error of children's placements, it does not allow inferences to be drawn about the sources of this error. The second method addresses this issue and provides an assessment of the pattern of placements across items by fitting *ln* and linear functions to median estimates and examining the goodness of fit of the corresponding regression equations (Siegler & Opfer, 2003; Siegler & Booth, 2004). Recall, use of the number-magnitude representational system will result in a pattern of placements that conforms to the *ln* of the numbers, whereas use of the school-taught linear representational system will result in a linear pattern of placements. Moreover, progress toward learning of the school-taught system and development of a linear representation of the number line can be assessed by examining how closely the linear regression model fits expected patterns; specifically, perfect linear placements—and mastery of the number line for the range assessed—would produce a regression equation with an intercept of 0 and a slope of 1.

The top portion of Figure 1 reveals that in first grade, the *ln* model ($r^2 = .86$) provided a better fit to the placements made by the MLD group than did the linear model ($r^2 = .65$), whereas the linear model provided better fits to the placements of both the LA ($r^2 = .94$ and .87 for linear and *ln* models, respectively) and TA ($r^2 =$ .97 and .89) groups. The bottom portion of Figure 1 reveals similar levels of fit for *ln* ($r^2 = .93$) and linear ($r^2 = .95$) models in second grade for the MLD group and very strong linear fits for the LA and TA groups (r^2s = .99).

Improvements in accuracy across grades can be assessed by changes in the intercept and slope values of the linear functions and by assessing how closely the values are to the respective expected values of 0 and 1. For all three groups, the intercept values changed significantly, $t(23)$s > 5.26, ps < .01, and were closer to the expected 0 value in second grade than in first grade. Similarly, all of the slope values changed significantly, $t(23)$s < –3.56, ps < .01, and were closer to the expected 1 value in second grade than in first grade. However, all second grade intercept terms remained greater than 0 (ps < .001) and all slope values differed (ps < .001) from 1. All pair-wise comparisons of intercept and slope values were significant (ps < .05), with the placements of the TA group being the closest to the expected values, followed in turn by the LA and MLD groups.

Whereas the second method is useful for examining group- and grade-level trends in the representational system that is most likely being used to make number line placements, it does not provide information on within-group differences in representational use or within-subject trial-by-trial variation in use of one representational system or the other. The third method addresses these issues and is a derivative of the second (Geary et al., 2007). The procedure involves calculating the absolute difference between each child's placement for each trial and the placement if they were using a linear or log representation. For the linear representations, we used the actual magnitude for the trial and for log representations we used the best fitting log equation found by Siegler and Booth (2004) for first-grade children's number line placements; $19ln(x) - 15$, where x = the value to be estimated. As an example, for "23," children making placements using a linear representation should place their estimates near 23, whereas children making placements using a log representation should place their estimates near 45. Siegler and Booth did not provide an *ln* equation for the second graders assessed in their study, and thus we used the values from the *ln* equation for MLD children (Figure 1), that is, $21ln(x) - 28$. The corresponding *ln* function results in more accurate placements than the first grade *ln* function, while maintaining the predicted placement pattern if children were using the natural number-magnitude system; the combination might emerge with natural maturation of the underlying brain and cognitive systems.

All trials were classified as linear or logarithmic based on whether the child's estimate was closer to the predicted value for the linear or log model. When the expected value for the linear and log models differed by less than ± 3 or the child's es-

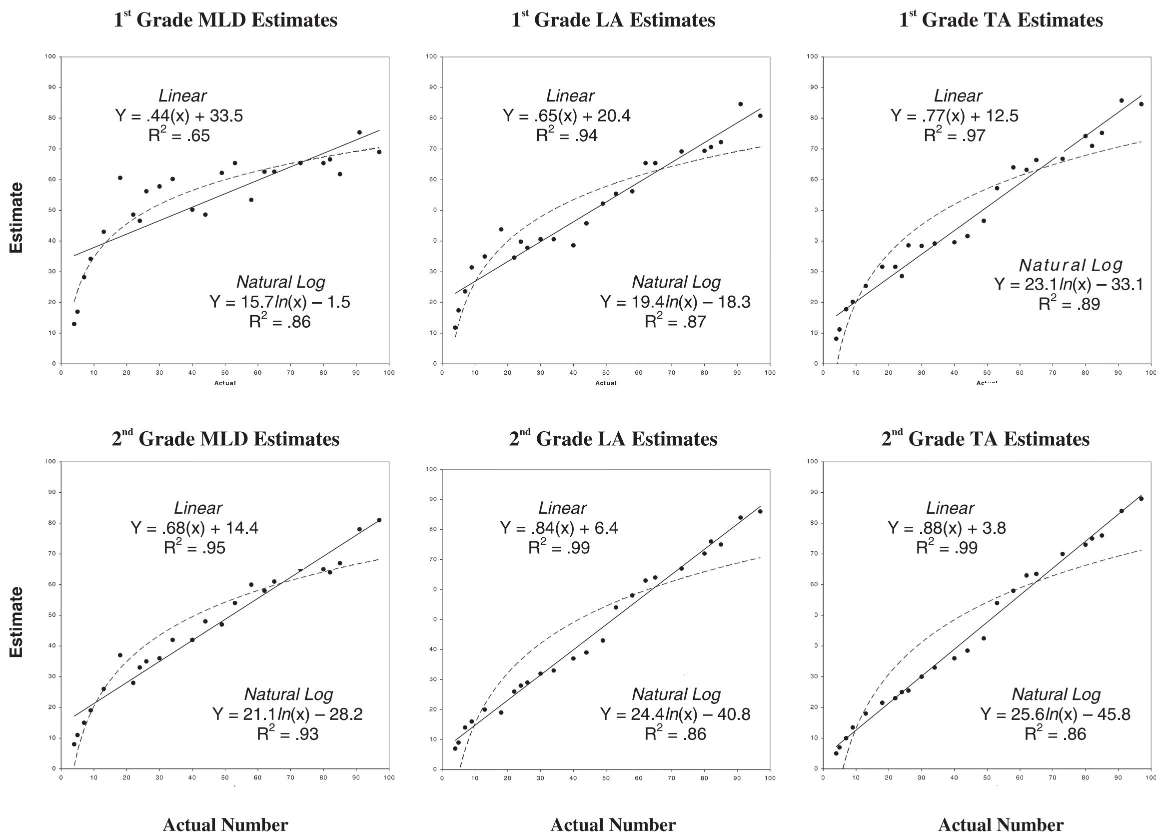

FIGURE 1 Linear and logarithmic fits to median number line placements in 1st (top) and 2nd (bottom) grades. MLD = math disabled, LA = low achieving, and TA = typically achieving.

timate was not clearly better fitted by either model, the trial was classified as ambiguous. Using this approach, we were able to make trial-by-trial classifications of whether each child was most likely to have used a linear or logarithmic representation to make each placement. The corresponding mean values are shown in Table 3.

For percent use of the linear strategy, a mixed ANCOVA, with IQ and second grade reading scores as covariates, revealed a significant group effect, $F(2, 107) = 7.10, p < .002$, but non-significant effects for grade and the group by grade interaction (ps > .25). LS t-tests revealed the MLD group used the linear strategy less frequently than the TA group in first ($d = -.90, p < .05$) and second ($d = -1.35, p < .001$) grade, and less frequently than the LA group in second grade ($d = -.95, p < .002$). None of the remaining contrasts were significant (ps > .05). When they used the linear strategy, the difference between the estimated and correct positions on the number line differed across groups, $F(2, 107) = 4.76, p < .02$, and there was a trend for a grade by group interaction, $F(2, 107) = 2.41, p = .095$, as shown in Figure 2. For first grade, the linear-strategy placements of the TA group were more accurate than those of the LA ($d = 0.40, p < .05$) and MLD ($d = 1.0, p < .01$) groups; the two latter groups did not differ ($p > .15$). For second grade, the linear-strategy placements of the MLD group were less accurate that those of the TA and LA groups (ds = 0.74, ps < .06).

It is also useful to examine how well the log strategy placements conform to the *ln* curve reported by Siegler and Booth (2004) for first graders. Examination of these placements is particularly important for first graders because all but one child in the TA group made at least some placements consistent with use of the number-magnitude representational system and because these placements are less likely to have been influenced by schooling than the second-grade placements. The mean difference between the log strategy placements and the predicted placements based on the *ln* curve was 8 ($SD = 6$), 13 ($SD = 8$), and 17 ($SD = 9$) for the TA, LA, and MLD groups, respectively, $F(2, 106) = 4.96, p < .01$. The log strategy placements of the TA group more precisely fitted the *ln* curve than did the placements of the LA, $t(106) =$

TABLE 3
Percentage of Children Using Linear and Logarithmic Strategies for Number Line Placements

	1st Grade Strategy			*2nd Grade Strategy*		
	Linear *M*	*Log* *M*	*Ambiguous* *M*	*Linear* *M*	*Log* *M*	*Ambiguous* *M*
MLD	24	69	7	37	52	11
LA	36	55	9	56	31	13
TA	42	47	11	64	23	13

Note. MLD = math disabled, LA = low achieving, and TA = typically achieving.

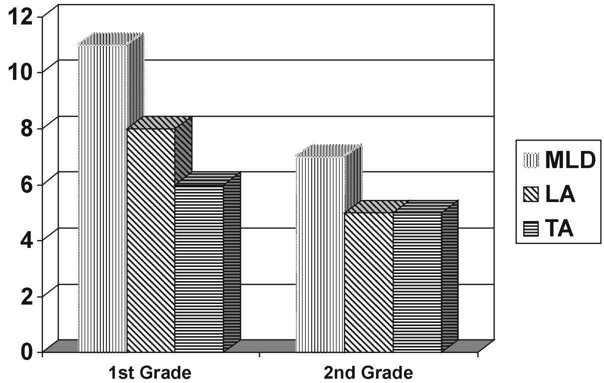

FIGURE 2 Mean degree of error when the linear representation was used to make the number line placements. MLD = math disabled, LA = low achieving, and TA = typically achieving.

–2.57, $p < .02$ ($d = 0.70$), and MLD, $t(106) = –2.80$, $p < .01$ ($d = 1.13$), groups; the two latter groups did not differ ($p < .25$). In other words, when the TA children used the log strategy to make number line placements, the placements were more coherently organized around the predicted natural number-magnitude system.

Working Memory and Number Line Development

To examine individual differences in accuracy of number line placements and the form of the underlying representations, we regressed overall degree of error, percentage use of the linear and log strategies, and accuracy of the linear-strategy placements on mean IQ, RAN RT, and the three working memory variables in each grade. For the second grade number line variables, we ran a second set of regression equations in which the corresponding number line variable from first grade was added to the set of predictors. These latter equations provide information on the potential sources of improvement from first to second grade, while controlling for initial level of competency in first grade.

For first grade, larger overall degree of error was associated with lower IQ scores, $\beta = –.39$, $t(97) = –4.38$, $p < .0001$, higher visuospatial sketchpad scores, $\beta = .30$, $t(97) = 2.85$, $p < .01$, and lower central executive scores, $\beta = –.28$, $t(97) = –2.26$, $p < .05$. More frequent use of the linear strategy was associated with higher IQ scores, ß = .46, $t(97) = 4.14$, $p < .0001$, whereas more frequent use of the log strategy was associated with lower IQ scores, $\beta = –.50$, $t(97) = –4.54$, $p < .0001$, and higher visuospatial sketch pad scores, ß = .23, $t(97) = 1.81$, $p = .074$. Accuracy of linear-strategy placements increased with increases in IQ, ß = .29, $t(97) = 2.91$, $p < .005$, and central executive scores, $\beta = .28$, $t(97) = 2.02$, $p < .05$, but decreased with increases in visuospatial sketchpad scores, $\beta = .34$, $t(97) = 2.97$, $p < .005$.

For second grade, larger overall degree of error was associated with lower IQ scores, ß = –.31, $t(97) = –3.23$, $p < .002$, and lower central executive scores, $\beta = –.42$, $t(97) = –3.12$, $p < .005$. More frequent use of the linear strategy was associated with higher IQ scores, $\beta = .50$, $t(97) = 5.12$, $p < .0001$, and higher central executive scores, $\beta = .37$, $t(97) = 2.76$, $p < .01$, whereas more frequent use of the log strategy was associated with lower IQ scores, ß = –.51, $t(97) = –5.24$, $p < .0001$, and lower central executive scores, $\beta = –.39$, $t(97) = –2.89$, $p < .005$. Accuracy of linear-strategy placements increased with increases in central executive scores, $\beta = .25$, $t(97) = 2.23$, $p < .05$. Rerunning these analyses while controlling for the corresponding first grade number-line variable revealed that higher central executive scores were associated with lower overall degree of error, $\beta = –.31$, $t(96) = –2.39$, $p < .02$, more frequent use of the linear strategy, $\beta = .35$, $t(96) = 2.59$, $p < .02$, less frequent use of the log strategy, $\beta = –.34$, $t(96) = –2.63$, $p < .01$, and more accurate linear-strategy placements, $\beta = .20$, $t(96) = 1.78$, $p = .078$. Higher IQ scores were associated with more frequent use of the linear strategy, $\beta = .40$, $t(96) = 3.90$, $p < .002$, and less frequent use of the log strategy, $\beta = –.37$, $t(96) = –3.64$, $p < .005$.

Cognitive Mediation

We assessed whether group differences on the phonological loop, central executive, and RAN RT variables mediated the group differences in overall degree of error, percentage use of the linear strategy, and accuracy of linear-strategy placements from the number line task; the visuospatial sketch pad was not included, because the group differences on this measure were not significant. We contrasted the MLD group with the LA and TA groups combined, because the two latter groups did not differ for two of the three potential mediators, that is, RAN RT and the central executive. IQ was included in all analyses, and for second grade the corresponding first grade number line variable was included. Initial analyses revealed the phonological loop and RAN RT measures were not significant predictors of the number line variables ($ps > .10$) and thus these were dropped from all further analyses. The final analyses included IQ, the central executive variable, and for second grade the corresponding first grade number line variable. Mediational effects were assessed following Baron and Keeney (1986) and using Sobel's (1988) test. If the mediational effect was significant and reduced the group contrast to nonsignificance ($p > .05$) then full mediation is implied; partial mediation if the group contrast remained significant.

For first grade, IQ emerged as a significant and partial mediator of the group differences in overall degree of error ($Z = 2.42$, $p < .02$) and percentage of linear trials ($Z = 2.38$, $p < .02$), and a trend emerged for full mediation of the group difference in linear-strategy accuracy ($Z = –1.69$, $p < .10$); the central executive did not emerge as a mediator. In second grade, IQ again emerged as a partial mediator of the group difference in percentage of linear trials ($Z = 2.44$, $p < .02$). For the central executive, trends emerged for partial mediation of the group difference in overall degree of error ($Z = –1.81$, $p <$

.08) and full mediation of the group difference in linear-strategy accuracy ($Z = -1.73$, $p < .10$). Unlike first grade, IQ was not a mediator of these two variables in second grade.

DISCUSSION

The current study is the first assessment of grade-related change in MLD and LA children's accuracy at making number line placements; the first study of change in the form of the underlying number line representational systems; and, the first analysis of individual and group differences in the cognitive mechanisms that might mediate these grade-related changes. We begin with discussion of group and individual differences in number line performance, and then consider the cognitive mechanisms that contribute to within- and between-grade differences comparing children with MLD to their LA and TA peers.

Number Line Representations

For all three of the groups assessed in the current study, number line performance was consistent with theoretical predictions and with previous empirical studies. Theoretically, the natural log and linear patterns that emerged for the median values (Figure 1) are consistent with respective use of natural number-magnitude and school-taught linear representations for making number line placements (Feigenson et al., 2004; Gallistel & Gelman, 1992; Siegler & Opfer, 2003). The procedure in which the children's estimates were fitted to the log and linear models on a trial-by-trial basis produced results that are also consistent with these predictions, but also suggested greater variation in children's use of one representational system or the other; specifically, for some trials children made placements that implicated use of a linear representation and for other trials they made placements that implicated use of the natural number-magnitude representation. With the exception of our earlier study (Geary et al., 2007), trial-by-trial variation in the strategies used to make number line placements has not been previously documented but is consistent with the more general pattern of trial-by-trial variation in children's problem-solving strategies in many cognitive domains (Siegler, 1996).

Group differences varied somewhat across the different methods used to analyze number line performance, but converged on a similar conclusion: Children with MLD were more heavily reliant on the natural number-magnitude representations to make their number line placements than were children in the LA and TA groups. Even when they made placements consistent with use of the natural number-magnitude system, the placements of children with MLD and their LA peers were less precise than those of the TA children in first grade, that is, before much if any formal instruction on the number line. The implication is that children with MLD and LA children may begin school with a less precise underlying system of natural number-magnitude representations. In first grade, the TA children used the linear strategy more frequently and were more accurate in when they used the linear strategy than were the LA children, but the frequency of linear strategy use and its accuracy increased for the children in both of these groups from first to second grade. By the end of second grade, the group differences were no longer significant on most measures. The children with MLD also showed across-grade improvements but at a slower rate than children in the two other groups.

Cognitive and Intellectual Mediators

Individual differences in number line performance—overall accuracy, use of the linear and log strategies, and linear-strategy accuracy—were related in various ways to individual differences in IQ, visuospatial working memory, and the central executive component of working memory. In first grade, IQ was particularly important, with higher scores related to more accurate overall placements, more frequent use of the linear strategy, and more accurate use of the linear-strategy. Higher central executive scores independently contributed to higher accuracy, overall and when the linear strategy was used. Although measures of IQ and the central executive are moderately to highly correlated (e.g., Ackerman, Beier, & Boyle, 2002; Conway, Cowan, Bunting, Therriault, & Minkoff, 2002), they also capture independent components of cognitive ability. Performance on measures of both the central executive and IQ require attentional and inhibitory control, but these mechanisms may be more important for performance on central executive measures than on IQ measures, especially if the latter are not timed (as with this study). Performance on measures of IQ, in contrast, is much more dependent on the ability to think logically and systematically (Embretson, 1995).

The relation between IQ and number line performance in first grade is entirely consistent with this aspect of intelligence: The ability to think logically and systematically facilitates learning the logical structure of the mathematical number line, that is, the base-10 organization of the numbers and the equal distance between successive numbers regardless of position on the line. Attentional and inhibitory control and working memory would be important for actual online, so to speak, placements of the numbers in accordance with the child's understanding of how numbers are represented across the line. We also found a seemingly paradoxical finding that higher visuospatial working memory was associated with lower overall accuracy, lower linear-strategy accuracy, and more frequent use of the log strategy. These results are not in fact paradoxical, if a strong visuospatial working memory system is associated with a well developed natural number-magnitude representational system (Zorzi et al., 2002). In this situation, children with a strong visuospatial working memory may represent numbers using the natural number-magnitude system more automatically than other children, but use of this natural system would result in less accurate placements—when measured against the linear standard—and more frequent use of the log strategy (Geary, 2007a), as we found.

In second grade, visuospatial working memory was no longer a predictor of number line performance, potentially because as children get older they become better at inhibiting the use of irrelevant or potentially interfering information (Welsh & Pennington, 1988). Both IQ and the central executive remained predictors of various aspects of number line performance in second grade, but across-grade changes were more consistently related to the central executive than to IQ. It is possible that once the logical structure of the number line is understood, the creation of a mental representation that conforms to this structure is more dependent on the attentional control and inhibitory components of the central executive than on IQ per se (Geary, 2005).

In any case, the children with MLD had average but still lower IQ scores than the TA children and substantially lower scores on the central executive than their peers in the LA and TA groups. The group differences in IQ and the central executive emerged as partial or full mediators of the comparatively poorer performance of the MLD children on the number line task, but to different degrees in first and second grade. In first grade, the small to moderate group differences in IQ contributed to the less accurate number line placements and less frequent use of the linear strategy by the MLD children. The implication is that these children are disadvantaged in the initial learning of the logical structure of the number line.

By second grade, the central executive emerged as a more consistent mediator of group differences than IQ. Both the pattern of median placements and the trial-by-trial strategies indicated the children with MLD were making linear placements more frequently in second than first grade, implying they were learning the logical structure of the number line and forming a mental representation of this linear structure. Their overall performance, however, indicated that their bias toward use of the log strategy remained. Their poor central executive skills might have contributed to the group differences in second grade in several ways. First, if the attentional control components of the central executive are important for the formation of a linear mental representation of the number line, then these children will be delayed in the formation of this representation and thus dependent on the natural number-magnitude representation for a longer period of time (Geary, 2007a). Second, the attentional control and inhibitory components of the central executive will be important for maintaining on-task focus when making number line placements and for inhibiting activation of the number-magnitude representational system. Brain imaging studies of number line performance and developmental change in this performance may help to clarify these potential mechanisms.

SUMMARY

The understanding of and ability to use the linear, mathematical number line is a core aspect of children's early mathematics education (National Council of Teachers of Mathematics, 2006), but has not been systematically studied in children with MLD or their LA peers. In this first longitudinal study on this topic, we found that children with MLD are less accurate than TA children in their placement of numbers on the number line, in part, because they appear to be more heavily dependent on the natural number-magnitude representational system to make these placements. From the beginning of first grade to the end of second grade, the children with MLD became more accurate in their placements and used the linear strategy more frequently but these changes were more modest in comparison with those that emerged with the TA and LA children. The initial, first grade, performance differences were related to a modest disadvantage in IQ for the MLD children, perhaps making the learning of the logical structure of the mathematical number line more difficult for these children. The disadvantage of the children with MLD in second grade and especially the slower rate of across-grade change was related to a substantive deficit in the central executive component of working memory. The latter deficit may slow the formation of a mental representation of the linear number line and may make the inhibition of the natural number-magnitude representation difficult when making actual placements on the number line. The LA children, in contrast, did not show this central executive deficit and by the end of second grade did not differ from TA children for most aspects of number line performance. The implication is that the relatively poor performance of the LA children, in comparison to that of the TA children, at the beginning of first grade appears to represent a mild developmental delay that responded to typical first and second grade instruction.

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