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Missouri longitudinal study of mathematical development and disability

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Background. Of all the students, 7% will experience a severe learning deficit in mathematics before completing high school. These students with a mathematical learning disability (MLD) are joined by another 5–10% of students with less severe learning difficulties.

Aims. The goal is to identify the core deficits that define MLD and mathematical difficulties; identify the cognitive systems that underlie these deficits; and, to develop measures to identify at-risk children.

Method. The Missouri project is a prospective kindergarten to 9th grade study of more than 200 students' mathematical growth and learning. Students are administered standardized achievement tests and mathematical cognition tasks once a year. A comprehensive working memory battery was administered in 1st and 5th grade, and classroom attention assessed in 2nd to 4th grade.

Results. Initial results indicate that children with MLD perform about 1 *SD* below average on working memory measures, even when IQ is controlled, and perform below average on mathematical cognition tasks that involve number processing and representation, execution of arithmetic procedures, and recall of arithmetic facts. Children with learning difficulties have average IQ and working memory test performance but are below average on number processing tasks and recall of arithmetic facts. Performance on 1st grade mathematical cognition tasks is predictive of MLD status in later grades.

Conclusions. Children with MLD have broad working memory deficits and specific deficits in their sense of number that delays their learning of formal mathematics. Children with learning difficulties do not have working memory deficits, but they do have a poor number sense and difficulties recalling arithmetic facts.

On the basis of several population-based, prospective studies and many smaller-scale studies, about 7% of children and adolescents will experience a substantive learning deficit – not attributable to low cognitive ability – in at least one area of mathematics before graduating from high school (Badian, 1983; Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Lewis, Hitch, & Walker, 1994;

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Ostad, 1998; Shalev, Manor, & Gross-Tsur, 2005). These are individuals with a mathematical learning disability (MLD), and are joined by another 5–10% of children and adolescents who will experience more mild learning difficulties in mathematics (Berch & Mazzocco, 2007). Individuals in this latter group are low achieving (LA), that is, their progress in mathematics is below expectations based on their cognitive ability and reading achievement but the factors underlying their difficulties are not as pervasive or severe as those that appear to underlie MLD (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Murphy, Mazzocco, Hanich, & Early, 2007).

The Missouri longitudinal study is part of a network of research labs funded by the National Institute of Child Health and Human Development (NICHD) to study mathematical learning and disabilities; http://www.nichd.nih.gov/research/supported/math.cfm. The primary goal of the Missouri study is to track the mathematical development of children with MLD and LA children from kindergarten to the completion of their first high school algebra course. The specific goals are to identify the mathematical areas in which children with MLD and LA children experience learning deficits and difficulties, respectively, and to identify the cognitive mechanisms that might underlie these deficits and difficulties. The longitudinal design will eventually allow for the identification of early predictors of long-term risk for MLD and LA and through this provide the foundation for the development of assessment measures and targets for early intervention for at risk children. In the first section, I describe the basics of the study. In the second and third sections, respectively, I focus on the core mathematical areas that are currently being assessed and the potential cognitive mechanisms that contribute to learning in these areas. In both of these latter sections, recent findings from the Missouri study are interleafed with the discussions.

Study overview

Design

The design of the study is shown in Table 1, and includes standardized assessments, mathematical tasks, cognitive tasks, psychometric tests, and a measure of classroom attention. The assessments through 5th grade have been completed; the children are currently in the 6th grade.

Standardized tests

The achievement tests are administered at the end of each academic year and include the numerical operations and word reading subtests from the *Wechsler individual achievement test-II – abbreviated* (Wechsler, 2001). At the end of 9th grade, all children are required by state law to pass an algebra competency test, and the score on this test along with the algebra course grade will be important long-term outcome measures. At the end of kindergarten, the children were administered the *Raven's coloured progressive matrices* (Raven, Court, & Raven, 1993), a non-timed test that is considered to be an excellent measure of fluid intelligence. At the end of 1st grade, the children were administered the vocabulary and matrix reasoning subtests of the *Wechsler Abbreviated Intelligence Scale* (Wechsler, 1999), and these scores were used to estimate IQ based on norms presented in the manual. The children's intelligence score is the mean of these two tests.

Table 1. Design of Missouri longitudinal study

		Grade								
Task	K	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
Standardized tests										
Mathematics achievement	X	X	X	X	X	X	X	X	X	X
Reading achievement	X	X	X	X	X	X	X	X	X	X
State algebra										X
Intelligence	X	X								
Mathematical tasks										
Number sets	X	X	X	X	X	X	X	X	X	X
Number line	X	X	X	X	X	X	X	X	X	X
Counting knowledge	X	X	X	X	X	X				
Arithmetic	X	X	X	X	X	X	X	X	X	X
Fractions							X	X	X	X
Cognitive tasks										
Working memory		X				X				
Speed of processing	X	X	X	X	X	X	X	X	X	X
Psychometric tests										
Arithmetical fluency							X		X	
Word problems							X		X	
Fractions							X		X	
Classroom										
Attention measure			X	X	X					

Mathematical tasks

The tasks were chosen based on areas in which children with MLD or LA children have been found to have deficits or difficulties in earlier studies (for reviews, see Berch & Mazzocco, 2007; Geary, 1993, 2004), or areas that are considered essential preparation (e.g. fractions) for algebra learning (National Mathematics Advisory Panel, 2008). The basics of the tasks are provided below, and detailed descriptions in Geary *et al.* (2007).

Number sets. We designed the *Number Sets Test* as a group-administered paper-and-pencil measure of the speed and accuracy with which children can identify the quantity of sets of objects – features of their early number sense (below) – and combine these with quantities represented by Arabic numerals (Geary *et al.*, 2007). Figure 1 shows several example items from the measure. There are four pages of such items, and the child is instructed to move across each line of the page from left to right without skipping any and to 'circle any groups that can be put together to make the top number, five (nine)' and to 'work as fast as you can without making many mistakes'. The child is given 60 and 90 s per page for the targets 5 and 9, respectively, and is asked to stop at the time limit. We chose to time the task to avoid ceiling effects and because a timed measure should provide an assessment of fluency in recognizing number combinations.

The task yields numbers of hits, misses, correct rejections, and false alarms for each problem type and size. Geary *et al.* (2007) found that 1st graders' performance was consistent across target number and item content (e.g. whether the rectangle included Arabic numerals or shapes) and could thus be combined to create an overall frequency of hits ($\alpha = .88$), correct rejections ($\alpha = .85$), misses ($\alpha = .70$), and false alarms ($\alpha = .90$).

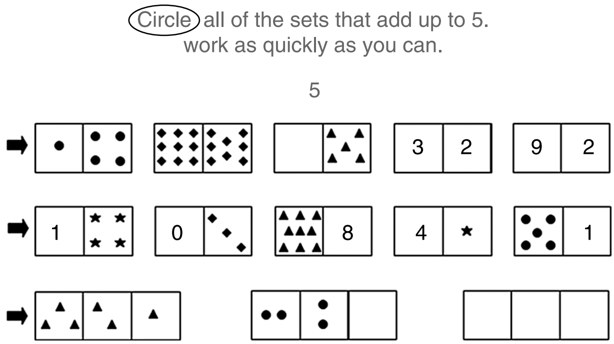

Figure 1. Example items from the *Number Sets Test*.

Number line estimation. We use the number line measure developed by Siegler and Opfer (2003). The stimuli are a series of number lines printed on paper or presented on a computer screen. Each number line has a start point of 0 and an end-point of 100 (or higher) and a target number printed above it. The child is asked to mark on the paper or use the mouse on the computer to indicate where the target number goes on the line. The target numbers were chosen following Siegler and Booth (2004, experiment 1) and allow for the fitting of logarithmic and linear regression models to the children's placements – the rationale is described below in the 'Number sense' section. The task provides several indices that can be used to make inferences about the cognitive representational system that children use to make number line placements and their progress in learning the linear mathematical number line. The indices are also predictive of later mathematics achievement (Booth & Siegler, 2006).

Counting knowledge. The goal is to assess children's knowledge of core principles of counting (Gelman & Gallistel, 1978) and the inductions they have made regarding essential and unessential features of counting (Briars & Siegler, 1984); these are elaborated below in the 'Counting' section. Children's knowledge of these counting principles and features is assessed by asking them to help a puppet learn how to count (Gelman & Meck, 1983); specifically, the child is introduced to a puppet that is just learning how to count and needs to know if his counting is okay and correct, or not okay and wrong. By using a puppet to do the counting, the task removes the need for children to engage in the procedural act of counting and thus should provide a less biased estimate of their emerging conceptual knowledge of counting.

There are a series of sets of items which the puppet sometimes counts correctly using the standard left-to-right procedure and sometimes counts correctly using an irregular procedure. An example of the latter involves the puppet counting the 1st, 3rd, 5th, and 7th items in the set from left to right and then returning to the left side and counting the 2nd, 4th, and 6th items and answering 'seven'. The count is technically correct but appears to be an error for children who do not understand a core counting principle or make incorrect inductions about essential features of counting. For other sets, the puppet violates a core principle. The assumption is that children who detect violations of basic principles will tell the puppet that these counts are not okay and wrong.

Addition strategy assessment. It is now clear that children use a mix of strategies when solving any series of academic or other type of problem (Siegler, 1996).

The strategy assessment task allows us to capture this variation during children's early learning of arithmetic, and provides several useful indices of learning progress and competence. Specifically, the task provides information about the sophistication of the mix of strategies used in problem solving, the accuracy of procedural execution – as in using a counting procedure to solve addition problems – and the accuracy of retrieving arithmetical information from long-term memory (Siegler, 1987; Siegler & Shrager, 1984). Performance on the corresponding indices is correlated with mathematics achievement (Geary & Burlingham-Dubree, 1989) and discriminates children with MLD and LA children from their typically achieving (TA) peers (e.g. Geary, 1990; Jordan & Montani, 1997).

In our task, children are presented with a series of simple (e.g. $5 + 8$) and more complex (e.g. $9 + 15$) addition problems to solve one at a time. For our strategy choice trials, the child is asked to solve each problem (without the use of paper-and-pencil) as quickly as possible without making too many mistakes and using whatever strategy is easiest get the answer. Beginning in 2nd grade, we added a series of retrieval–only simple addition problems and instruct the children to only use 'remembering' to get the answer. These 'forced retrieval' trials allow for a more detailed assessment of the retrieval deficit (below) that is a cardinal aspect of MLD (e.g. Geary, Hamson, & Hoard, 2000; Jordan & Montani, 1997).

While the child is solving each problem, the experimenter watches for physical indications of counting, such as regular finger or mouth movements. These trials are initially classified as finger counting or verbal counting, respectively. On verbal counting trials, the experimenter probes the child as to how she counted, and the child's response is recorded as min, sum, or max. Min involves stating the larger valued addend and then counting a number of times equal to the value of the smaller addend (e.g. counting 3, 4, 5 to solve $3 + 2$), and sum involves counting both addends starting from 1. The max procedure involves stating the smaller addend and then counting the larger one. Finger counting trials are coded in the same way.

If the child speaks the answer quickly, without hesitation, and without obvious counting-related movements, then the trial is initially classified as direct retrieval of the answer or as decomposition if this was the child's predominant retrieval-based strategy on previous trials. An example of decomposition is provided by the problem, $18 + 7$, whereby the 7 can be decomposed into a set of 5 and a set of 2, followed by $18 + 2$ and then $20 + 5$. After the child states the answer, the experimenter queries her on how she got the answer. We have found good agreement between the experimenter's observations and children's responses, but sometimes there is a disagreement. If counting is overt, the trial is classified as a counting strategy. If the process that resulted in the answer is ambiguous, the child's response is recorded as the strategy.

Fractions. For fractions, basic computational and conceptual skills will be assessed using modified versions of Hecht, Close, and Santisi's (2003) and Mazzocco and Devlin's (2008) tasks.

Working memory and speed of processing

As noted, a core goal of the Missouri study is to identify the cognitive mechanisms that underlie the deficits of children with MLD and LA children on the mathematical tasks.

Our focus is on Baddeley and Hitch's (1974) three core working memory systems – the central executive, phonological loop, and visuospatial sketch pad – and on speed of processing; the rationale is provided in 'Cognitive mechanisms' section. Working memory is assessed using the nine subtest working memory test battery for children (Pickering & Gathercole, 2001). Speed of processing is assessed using two rapid automatized naming tasks (Denckla & Rudel, 1976; Mazzocco & Myers, 2003). These require children to state a series of letters and numbers as quickly and accurately as possible, and provide reliable reaction time measures.

Psychometric measures

As shown in Table 1, the psychometric tests will assess the children's developing computational fluency, their ability to solve multi-step arithmetical word problems, and their competence in dealing with fractions; specifically, their procedural skills in solving fraction problems and their conceptual understanding of them. These are all important competencies in and of themselves and are critical for children's preparation for algebra (National Mathematics Advisory Panel, 2008). The measures will include paper-and-pencil tests from the Educational Test Service *Kit of factor referenced tests* (Ekstrom, French, & Harman, 1976). These assess computational arithmetic and conceptual and procedural competence when solving arithmetical word problems.

Classroom behaviour

Classroom behaviour is assessed using the SWAN measure of classroom attention (strengths and weaknesses of ADHD-symptoms and normal-behaviour scale; Swanson *et al.*, 2008). The measure includes items that assess attentional deficits and hyperactivity but the scores are normally distributed, based on the behaviour of a typical child in the classroom. The measure was added to the Missouri study based on the finding that classroom attention contributes to arithmetical learning above and beyond the attentional control assessed by central executive measures of working memory (Fuchs *et al.*, 2006). The child's classroom teacher is asked to rate the behaviour of the child. We have collected data for a substantial proportion of the sample in 2nd, 3rd, and 4th grade. The analyses have not yet been completed and thus will not be discussed further.

Sample

All kindergarten children from 12 elementary schools were invited to participate; the schools serve children from a wide range of socio-economic backgrounds, but include several schools that have had a high proportion of children with MLD in previous studies. Parental consent and child assent were received for 37% ($N = 311$) of these children, and 305 of them completed the first round of testing at the end of kindergarten; 238 remained in the study at the end of 5th grade. The sample mean ($M = 99$) and standard deviation ($SD = 15$) on the nationally normed *Wechsler Abbreviated Intelligence Scale* were both at the national average. At the end of 1st grade, the mean reading achievement score ($M = 106$) and standard deviation ($SD = 16$) were slightly above the national average. Consistent with inclusion of schools with a higher proportion of children with MLD in previous studies, the mean mathematics achievement score ($M = 92$) and standard deviation ($SD = 13$) were below the national average.

Identifying children with MLD and LA children

Following Murphy *et al.* (2007), we have used more and less restrictive criteria to identify groups of children with MLD and LA children, respectively (Geary *et al.*, 2007). Because there are no readily available standardized measures specifically designed to diagnose MLD, most researchers rely on standardized achievement tests, sometimes in combination with IQ measures. Children with low average or higher IQ scores can score poorly on mathematics achievement tests for reasons other than a learning disability, and thus a low mathematics achievement score in and of itself cannot be considered an indicator of MLD. We have found that children with low achievement scores in one grade but higher scores in another do not typically have a cognitive deficit that would indicate MLD. However, children who score poorly on mathematics achievement tests across two successive grades often do show deficits on one or more of the mathematical tasks described above (Geary, 1990; Geary, Brown, & Samaranayake, 1991; Geary, Hoard, & Hamson, 1999; Geary *et al.*, 2000).

In our initial analysis, we classified children as MLD if their national ranking was equal to or less than the 15th percentile in both kindergarten and 1st grade and their IQ was between 80 and 130; Murphy *et al.* (2007) used similar criteria. The use of this cut-off across successive grades identified 5.4% of our sample with mean mathematics achievement scores at the 8th and 6th national percentile in kindergarten and 1st grade, respectively. The achievement scores and percentage of the sample identified as MLD are consistent with prevalence estimates obtained with restrictive criteria used in Barbaresi *et al.*'s (2005) prospective study of MLD. Children with percentile rankings between 23 and 39 on the mathematics achievement test in either grade were identified as LA (38 had rankings < 39th percentile in both grades), and provided a comparison group similar to samples identified as MLD in most previous cognitive studies.

Mathematical cognition

Number sense

A fundamental aspect of children's number sense is an implicit understanding of the absolute and relative magnitude of sets of objects, and of symbols (e.g. Arabic numerals) that represent the quantity of these sets. Children's early number sense includes an ability to immediately apprehend (without counting) the numerical value associated with sets of 3–4 objects or actions (Starkey & Cooper, 1980: Strauss & Curtis, 1984; Wynn, Bloom, & Chiang, 2002); a facility with use of counting to quantify small sets of objects and to add and subtract small quantities to and from these sets (Gelman & Gallistel, 1978; Starkey, 1992); and a proficiency in approximating the magnitudes of small numbers of objects and simple numerical operations (Dehaene, 1997). This intuitive sense of quantity and magnitude may be inherent (Butterworth & Reigosa, 2007; Dehaene, Piazza, Pinel, & Cohen, 2003; Geary, 1995) and may provide the foundation for early mathematics learning in school (Geary, 2006). On the basis of our previous work and the work of others (Siegler & Booth, 2004; Siegler & Opfer, 2003), we are focusing on two aspects of basic number sense: the speed and accuracy of identifying and processing number sets and the ability to represent quantity along a mathematical number line.

Number sets

In addition to being a core aspect of number sense, a conceptual understanding of sets and the ability to manipulate them in accordance with mathematical principles are critical competencies in academic mathematics. Children with MLD have potential deficits in both the core ability to apprehend the quantity of small sets and in the conceptual insight that numbers are composed of sets of smaller numbers that can be manipulated in ways that can facilitate mathematical problem solving.

With respect to the former, Koontz and Berch (1996) assessed the ability to apprehend, without counting, the quantity of small sets of objects or Arabic representations of these sets (e.g. 3 = ■■■). In this study, 3rd and 4th grade children with MLD and TA children were administered a variant of the physical identity and name identity task (Posner, Boies, Eichelman, & Taylor, 1969). Children were asked to determine, as an example, if combinations of Arabic numerals (e.g. 3–2), object sets (e.g. ■■ – ■■), or numerals and sets were the same (e.g. 2 – ■■) or different (e.g. 3 – ■■). In keeping with previous studies (Mandler & Shebo, 1982), reaction time patterns for the TA children indicated fast access to representations for quantities of two and three, regardless of whether the code was an Arabic numeral or number set. The children with MLD showed fast access to numerosity representations for the quantity of two, but appeared to rely on counting to determine quantities of three. The results suggest that some children with MLD might not have an inherent representation for numerosities of three or the representational system for three does not reliably discriminate two from three.

With respect to the latter, Geary, Hoard, Byrd-Craven, and DeSoto (2004) hypothesized that knowledge of number sets facilitates use of problem solving strategies that depend on the decomposition of number sets; the decomposition strategy (e.g. $18 + 7 = 18 + 2 + 5$) is sometimes used by TA children to solve simple and complex addition problems and is frequently used by intellectually precocious children (Geary *et al.*, 2004; Hoard, Geary, Byrd-Craven, & Nugent, 2008). In our initial study from the Missouri project, we found slower and less accurate processing of number sets by children with MLD and, to a lesser extent, LA children, and that children in these groups almost never used the decomposition strategy (Geary *et al.*, 2007). These results are suggestive of a relation between fluency of processing number-set information and use of decomposition to solve arithmetic problems, but are far from definitive.

In a follow-up analysis, Geary, Bailey, and Hoard (2009) used signal detection methods to determine the diagnostic utility of beginning of 1st grade scores on the *Number Sets Test* for predicting MLD status at the end of 3rd grade. The signal detection analysis provides two key variables, sensitivity (d′) and response bias (C; MacMillan, 2002). The former represents the child's sensitivity in the detection of quantities represented in task items and the response bias represents the child's tendency to respond to task items, whether they are correct or not. Children who correctly identify many target quantities and make few false alarms (i.e. circle an item that does not match the target quantity) and will have high d′ and low C scores, whereas children who have as many hits as false alarms will have low d′ and high C scores. In the latter case, the high number of correct items is due to the child's bias to respond and not sensitivity to quantity.

Higher d′ scores were associated with higher mathematics achievement (ps $< .001$) in kindergarten ($r = .58$), 1st grade ($r = .50$), 2nd grade ($r = .52$), and critically 3rd grade ($r = .49$), whereas the correlations between C scores and achievement were less consistent and the critical correlation with 3rd grade mathematics achievement scores ($r = -.14$) indicated little predictive value of C above and beyond d′. Further analyses indicated that 1st grade d′ scores were predictive of 3rd grade mathematics achievement, above and beyond the influence of 1st grade mathematics achievement scores, IQ, and working memory. In contrast, d′ scores were not predictive of 3rd grade reading achievement, indicating that this measure is tapping competencies unique to mathematics.

In keeping with Geary *et al.* (2007) and Murphy *et al.* (2007), children were classified as having MLD if they scored below the 15th percentile on *both* the 2nd and 3rd grade mathematics achievement test. Corresponding categories were then created based on 1st grade mathematics achievement scores and d′ scores; students scoring below the 15th percentile on the 1st grade mathematics test or below the 15th percentile of our sample on the d′ measure were diagnosed as at risk for MLD. The diagnostic utility of the achievement test and d′ was assessed in terms of sensitivity and specificity (Altman & Bland, 1994). Sensitivity is the ratio of true positives (hits) to total positives (hits + false alarms), and specificity is the ratio of true negatives (correct rejections) to total negatives (correct rejections + misses). Using these gross (i.e. 15th percentile) cut-offs, the specificity of both measures was high – 96% of children who did not have MLD in 3rd grade were correctly identified as non-MLD in 1st grade. The sensitivity – the percentage of 3rd grade children with MLD correctly diagnosed in 1st grade – of d′ (51% correctly identified) was higher than that of 1st grade mathematics achievement scores (40% correctly identified), but neither was particularly high.

We then used response operator curves to maximize sensitivity and specificity of the 1st grade mathematics achievement and d′ scores in predicting 3rd grade MLD; that is, to determine the optimal trade off between sensitivity and specificity. The procedure allowed us to determine cut-off scores for d′ that correctly identify 2 out of 3 of the 3rd grade MLD children and correctly exclude 9 out of 10 of the non-MLD children. The 1st grade achievement test scores also correctly identified 2 out of 3 of the 3rd grade MLD children but the same cut-off incorrectly identified 1 out of 3 of the non-MLD children as MLD. In terms of practice, the optimal cut-off values depend on the costs and benefits of early identification and remediation. If remediation is inexpensive and easily achieved, then maximizing sensitivity, regardless of changes in specificity, is likely to be the best approach. This is because the majority of children at risk for MLD will be identified and provided remedial services, and the costs of providing these services to children who do not need them is small. With limited resources, however, sensitivity must be balanced against specificity so that resources can be most effectively used. In any case, the *Number Sets Test* is not yet ready for use as a formal diagnostic instrument but it does show promise as a potentially quick (~10 min), group-administered screening measure.

Number line

Learning the mathematical number line (e.g. the difference between two consecutive numbers is identical regardless of position on the number line) is a core element of basic education in mathematics (Case *et al.*, 1996; Griffin, Case, & Siegler, 1994), and is an area of active study in cognitive psychology (Siegler & Booth, 2004; Siegler & Opfer, 2003) and cognitive neuroscience (Zorzi, Priftis, & Umiltá, 2002). As noted earlier, individual differences in children's learning of the linear, mathematical number line are correlated with mathematics achievement in all grades in which it has been assessed (Booth & Siegler, 2006). Children's competence with the number line is also of theoretical interest because magnitude representations, including those that support the mathematical number line, may be based on a potentially inherent number-magnitude system that is supported by specific areas in the parietal cortices (Isaacs, Edmonds, Lucas, & Gadian, 2001; Kadosh *et al.*, 2007; Molko *et al.*, 2003).

Making placements on a physical number line that are based on use of the inherent number-magnitude system results in a pattern that conforms to the natural logarithm (ln) of the number (Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 1992; Siegler & Opfer, 2003); use of this representational system results in placements that are compressed for larger magnitudes such that the perceived distance between 89 and 90 is smaller than the perceived distance between 2 and 3. When making number line placements TA children initially rely on the natural representational system, but quickly learn the linear system with schooling (e.g. Siegler & Booth, 2004). If children with MLD have deficits in the number-magnitude representational system (Koontz & Berch, 1996), then their number line placements might not conform to the natural log model or might show less precision than other children when they make placements using this representation; specifically, more compression (closer placements) for smaller numbers. If children with MLD do not show evidence of a developing linear representational system, then another source of MLD might be difficulty in modifying the natural system to conform to the school-taught linear system.

With respect to these issues, we have found several potentially important patterns in the number line placements of children with MLD and LA children (Geary, Hoard, Nugent, & Byrd-Craven, 2008; Geary *et al.*, 2007). As found by Siegler and his colleagues (Siegler & Booth, 2004; Siegler & Opfer, 2003), the placements of TA children on the number line were linear; that is, the placements were consistent with the learning of the linear property of the formal mathematical number line. The placements (median values) of our groups of MLD and LA children differed from each other and from those of TA children. As shown at the top of Figure 2, the children with MLD made placements consistent with reliance on the natural number-magnitude system (represented by the ln), whereas the placements of the LA children were consistent with use of this system for smaller numbers and use of a linear representation for larger numbers. The LA children showed a clear shift to use of the linear system by 2nd grade and, in fact, their 2nd grade performance did not differ substantively from that of the 2nd grade TA children. The performance of the MLD children in 2nd grade was similar to that of the LA children in 1st grade.

We also fitted the log and linear models on a trial-by-trial basis for each child. The group differences in use of the log and linear representations to make each placement on the number line were consistent with the pattern shown in Figure 2, but also suggested greater trial-by-trial variation in children's use of one representational system or the other; specifically, for some trials children made placements that implicated use of a linear representation and for other trials they made placements that implicated use of the natural number-magnitude representational system. Group differences varied somewhat across the different methods used to analyse performance, but converged on a similar conclusion: children with MLD were more heavily reliant on the natural number-magnitude representational system to make their number line placements than were children in the LA and TA groups. Even when they made placements consistent with use of the natural number-magnitude system, the placements of children with MLD and their LA peers were less precise than those of the TA children early in 1st grade, that is, before much if any formal instruction on the number line. By 2nd grade, LA children caught up with their TA peers, but the MLD children, though they improved, lagged behind the other children.

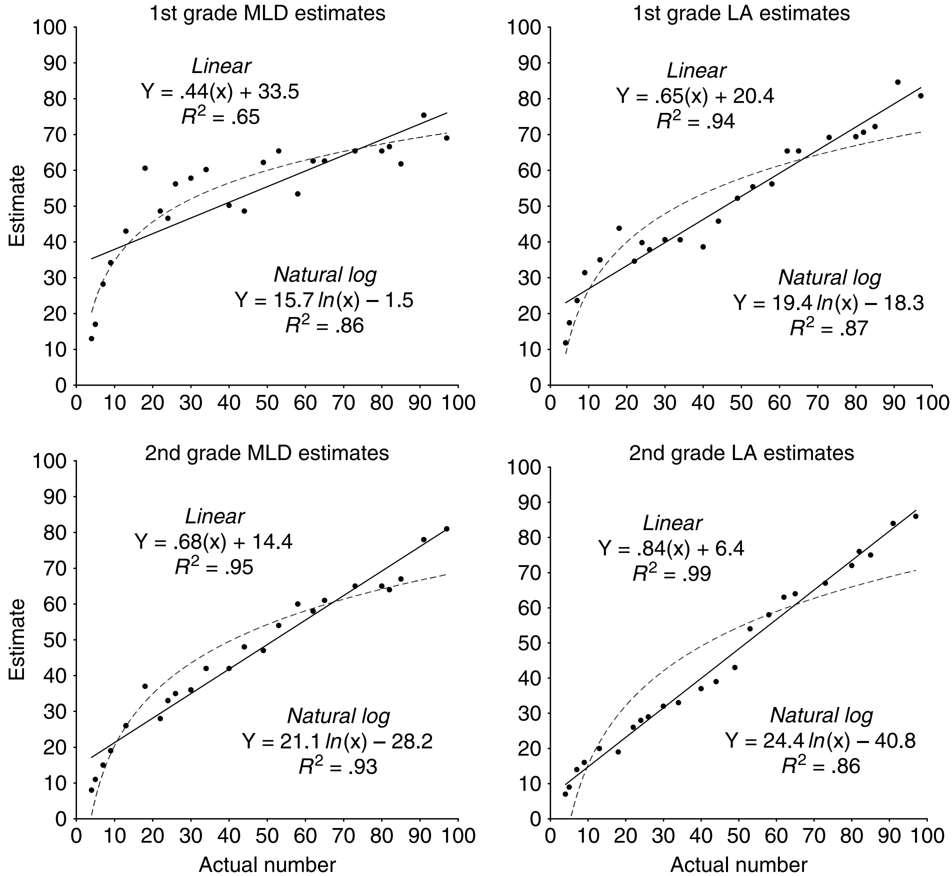

Figure 2. Linear and logarithmic fits to median number line placements in 1st (top) and 2nd (bottom) grades. MLD, mathematical learning disability; LA, low achieving.

One possibility is that children with MLD and LA children begin school with a less precise underlying system of natural number-magnitude representations. This system may quickly mature – leading to an improved ability to discriminate between quantities of near equal value – in LA children and may not mature or do so at a slower rate for children with MLD (see also, Halberda, Mazzocco, & Feigenson, 2008). We also found evidence that the central executive deficit of children with MLD (below) may slow their ability to mentally construct or use the school-taught linear system. Our longitudinal design will allow us to better address each of these possibilities.

Counting

By the time TA children enter kindergarten, they understand most basic counting concepts and can use counting procedures in many problem-solving contexts (Briars & Siegler, 1984; Gelman & Gallistel, 1978). The basic concepts include Gelman and Gallistel's five implicit principles; one-to-one correspondence (one and only one word tag is assigned to each counted object); stable order (the order of the word tags must stay the same across counted sets); cardinality (the value of the final word tag represents the quantity of items in the set); abstraction (objects of any kind can be collected together and counted); and order-irrelevance (items within a given set can be tagged in any sequence). Children's counting behaviour suggests they also make inductions about counting rules (Briars & Siegler, 1984): young children infer that the unessential features of adjacency (items must be counted contiguously) and start at an end (counting must start on the left) are essential.

Several important results regarding children's implicit knowledge of counting principles and their inductions about counting have emerged from the use of the counting knowledge task or a variant of it. The first is that children with MLD and LA children understand most of the counting principles proposed by Gelman and Gallistel (1978) but, second, they often make errors on items that assess order-irrelevance or adjacency (Geary, Bow-Thomas, & Yao, 1992; Geary *et al.*, 2004). These are counts that are correct but the procedure is executed in an irregular way, as described earlier. For kindergarteners to 2nd graders, LeFevre *et al.* (2006) found a curvilinear pattern for performance on these types of irregular counts. Paradoxically, they found that children with low numerical test scores tended to say these counts were correct in kindergarten and 1st grade, whereas their high ability peers tended to say these counts were incorrect. The pattern reversed in 2nd grade. LeFevre *et al.* hypothesized that children have an initial bias to state all counts are correct, except for those with obvious errors, and that an early awareness of variation in use of counting procedures leads to rejection of irregular but correct counts, and thus an initial disadvantage for high ability children. With experience, children eventually understand that irregular ways of counting do not violate core principles (e.g. cardinality) and at this point they accept these counts as correct.

If LeFevre *et al.*'s (2006) hypothesis is correct, then young children with MLD or LA children should perform better on these irregular, pseudo-error counts than TA children; not because they understand counting better but because they are less likely to notice that it is an irregular count and thus state the default 'correct'. The results from our cross-sectional studies are mixed in this regard (Geary *et al.*, 1992, 2004, 2007). Sometimes children with MLD perform 'better' on these items and sometimes worse. Geary *et al.* (2007) found that 1st grade children with MLD performed better – identifying pseudo errors as correct – than their LA or TA peers and Geary *et al.* (2004) found that 3rd and 5th children with MLD performed more poorly than their TA peers. The overall pattern suggests, as predicted by LeFevre *et al.*'s (2006) model, that at least some children with MLD are delayed in their attentiveness to variation in use of counting procedures and delayed in the inductions they make regarding the meaning of these variations. Our longitudinal design will allow for a stronger test of LeFevre *et al.*'s hypothesis as it relates to MLD.

We have also found that children with MLD, but not necessarily LA children, fail to detect errors when the first item is double counted (i.e. the item is tagged 'one', 'two'), but they detect these double counts when they occur with the last item. The pattern suggests children with MLD understand one-to-one correspondence, but have difficulty retaining a notation of the counting error in working memory during the count (Geary *et al.*, 2004; Hoard, Geary, & Hamson, 1999). Ohlsson and Rees (1991) predicted that children's counting knowledge and skill at detecting counting errors would enable them to correct these miscounts and thus eventually commit fewer errors when using counting to solve arithmetic problems. In support of this prediction, Geary *et al.* (1992) found that frequent failures to detect double-counting errors and to state pseudo-errors were incorrect were associated with frequent errors when counting was used to solve simple addition problems. The performance on the counting task also explained the higher frequency of counting procedure errors on the addition task for a group of children with MLD as compared to a control group of TA peers. A follow-up study revealed a more nuanced pattern; for 1st graders, the tendency of children with MLD to commit more errors when using counting to solve addition problems was more strongly related to IQ and working memory than to counting knowledge, but counting knowledge was more important for 5th graders. Whatever the reason, early performance on counting knowledge tasks appears to be a useful indicator of later risk for MLD (Gersten, Jordan, & Flojo, 2005).

Arithmetic

Children use a mix of counting and memory-based processes to solve simple addition problems (Ashcraft, 1982; Siegler & Shrager, 1984). The mix is initially dominated by finger and verbal (e.g. out loud) counting and the most commonly used procedures are sum and min (Fuson, 1982; Groen & Parkman, 1972). As a result of schooling and practice, children use the min procedure more often and eventually rely primarily on decomposition and retrieval.

In comparison to TA children, children with MLD rely on finger counting for more years, adopt the min procedure at a later age, and commit more counting errors (Geary, 1993; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Montani, 1997; Ostad, 1997). The most consistent finding is that children with MLD show a deficit in the ability to use retrieval-based processes (Barrouillet, Fayol, & Lathuliére, 1997; Geary, 1990; Geary *et al.*, 2000; Jordan, Hanich, & Kaplan, 2003). It is not that these children never correctly retrieve answers. Rather, they show a persistent difference in the frequency with which they correctly retrieve basic facts, and sometimes in the pattern of retrieval errors. We confirmed this pattern with our initial analysis of the Missouri cohort (Geary *et al.*, 2007) and will be following the development of the ability of MLD children to learn basic facts and testing hypotheses regarding the potential sources of their deficit in this area.

Cognitive mechanisms

Working memory

Working memory is the ability to hold a mental representation of information in mind while simultaneously engaging in other mental processes. The central executive is expressed as attention-driven control of information represented in two core representational systems (Baddeley, 1986). The systems are a language-based phonetic buffer and a visuospatial sketch pad. Baddeley (2000) has also proposed the existence of a third representational system, the episodic buffer, but measures of this system are not currently available and thus not included in our study.

We have hypothesized that the central executive is important for the initial stages of academic learning, that is, for the acquisition of novel school-taught competencies (e.g. linear number line) and suppression of more natural modes of understanding the presented information (e.g. use of the natural number-magnitude system; Geary, 2007). The central executive should contribute to performance on all of our mathematical tasks. We have predicted the visuospatial sketch pad and phonological loop will contribute to learning in more restricted mathematical domains. The visuospatial sketch pad, for instance, is of theoretical interest because the parietal areas associated with number and magnitude processing are situated near brain regions that support aspects of visuospatial processing and because damage to these parietal regions disrupts the ability to form spatial representations and to imagine a mental number line (Zorzi *et al.*, 2002). In theory, the visuospatial sketch pad should contribute to performance on the *Number Sets Test* and the number line task, and we have found supportive evidence for both tasks (Geary *et al.*, 2007, 2008).

The working memory systems in fact interact in complex ways during mathematics learning. Geary *et al.* (2008) found that a strong visuospatial working memory was associated with more frequent use of the natural number-representational system to make number line placements. This makes sense, theoretically, but results in poorer performance if the goal is to learn the linear mathematical number line. Learning to use a linear representation to make number line placements was related to IQ in 1st grade and the central executive in 2nd grade. We suggested that IQ contributes to children's ability to learn the logical structure of the mathematical number line, that is, the base-10 organization of the numbers and the equal distance between successive numbers regardless of position on the line. And, the central executive contributes to on-line performance during the placements, in part through inhibition of the natural number representational system.

In any case, Geary *et al.* (2007) found a substantial (~ 1 *SD*) deficit for average-IQ children with MLD for all three working memory systems, but no such deficit for LA children. As found with other studies, the central executive appeared to be one of the mechanisms that contributed to many of the above-described mathematical cognition deficits of the children with MLD (McLean & Hitch, 1999; Swanson, 1993; Swanson & Sachse-Lee, 2001). The deficit of the MLD children for the visuospatial sketch pad contributed to their poor identification of quantity (i.e. hits) on the *Number Sets Test* and their poor phonological loop contributed to frequent errors when using a counting procedure to solve addition problems. We will continue to explore these relations and others (e.g. Geary *et al.*, 2004) in the Missouri study.

Speed of processing

The potential contributions of working memory to mathematics learning are complicated by speed of processing. The issue is whether individual differences in working memory are driven by more fundamental differences in speed of neural processing (Kail, 1991), or whether the attentional focus associated with the central executive speeds information processing (Engle, Tuholski, Laughlin, & Conway, 1999). Either way, a systematic assessment of the potential mechanisms underlying the deficits of children with MLD and LA children requires simultaneous measurement of working memory and speed of processing.

Indeed, we have found slower information processing for our MLD and LA groups in comparison to their TA peers (Geary *et al.*, 2007). By simultaneously estimating the contributions of working memory and speed of processing to individual and group differences on the mathematical cognition tasks, we have also found working memory (especially the central executive) is the stronger candidate as a core mechanism underlying these individual and group differences than speed of processing.

Conclusion

For decades, our understanding of mathematical learning and learning disabilities lagged our understanding of reading and reading disabilities, but much has changed in the past 10 years. Advances in the understanding of the mathematical thinking and learning of TA children has provided a strong theoretical foundation and many experimental methods that can be applied to the study of MLD and LA (see Campbell, 2005). We have taken advantage of these advances in the design of the Missouri longitudinal study and are focused on identifying the areas of mathematics that are of particular difficulty for children with MLD and LA children, and on better understanding the sources of their respective disabilities and difficulties in these areas. In combination with the other labs supported by the NICHD consortium, as well as contributions from other researchers, we are poised in the next 10 years to make substantial progress in our understanding of the nature and sources of MLD and LA and in the development of assessment measures and remedial techniques (see Berch & Mazzocco, 2007).

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